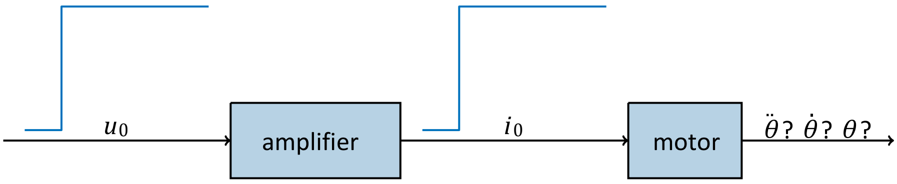
Exercise Set 8 - Electric actuators - Solutions

Exercise 1

Consider a DC motor with:

- J_m the inertia of the motor.
- R_a the winding resistance.
- L_a the winding inductance.
- k_e the electrical constant
- k_t the torque constant

This motor is controlled in current by a voltage-current amplifier.

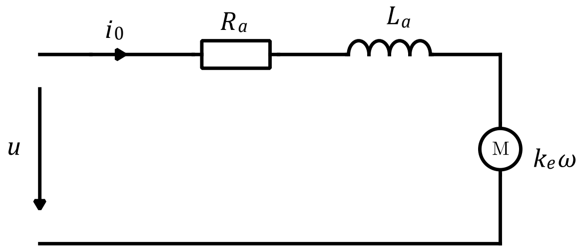


Give the shape of the acceleration, speed, and position curves for a step input i_0.

Hint: First derive the theoretical response to the given input. Later, derive the realistic response by taking into consideration that the amplifier's output voltage is limited by the supply voltage.

Exercise 1 - Solution

The equivalent model of the DC motor is:



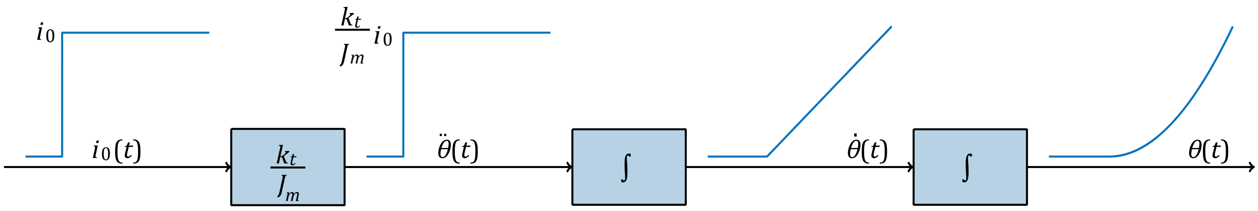
Its electrical equation is:

$$u = R_a i_0 + k_e \omega + L_a \frac{di_0}{dt}$$

Its dynamic (mechanical) model is:

$$J_m \ddot{\theta} = k_t i_0 \Rightarrow \ddot{\theta} = \frac{k_t}{J_m} i_0$$

From this dynamic model, we derive the following theoretical response to a current step:

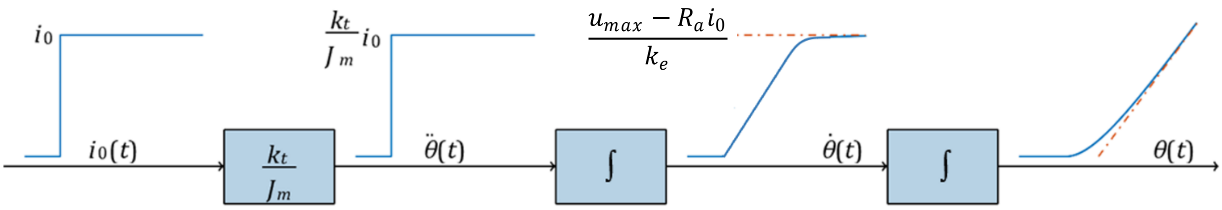


The responses in speed and acceleration are unrealistic because they give an infinite speed $\dot{\theta}$ for a time t sufficiently large.

However, the speed will not reach an infinite value because it is related to the voltage in the armature of the motor. The more the speed increases, the more this voltage in the armature increases, and this voltage is limited by what the amplifier can provide. Consider u_{max} to be the maximum voltage across the motor armature supplied by the amplifier, which is itself limited by the amplifier's power supply.

The electrical equation becomes:
$u = u_{max} = R_a i_0 + k_e \omega_{max}$ when the current is constant, which gives us the following maximum speed:

$$\omega_{max} = \frac{u_{max} - R_a i_0}{k_e}$$



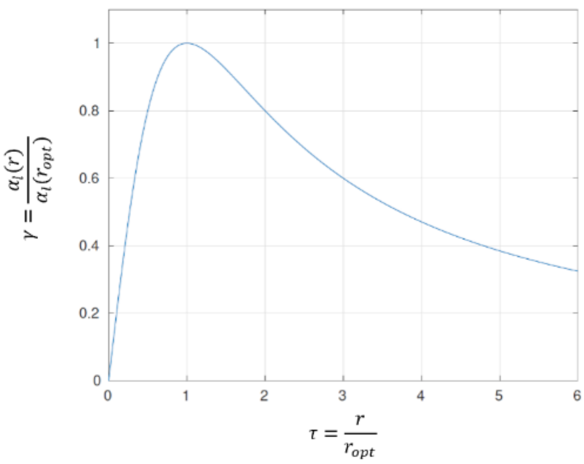
This means that the motor goes up to the maximum speed for any index entry i_0 that lasts long enough. This shows why the Ziegler Nichols open loop method is not used to set the PID parameters for position control.

Exercise 2

The motor axis of the second rotation of a SCARA robot is produced by the combination of a motor and a reducer with the following specifications:

- motor inertia: $J_m = 625$ g.cm^2
- reduction ratio: $n = 180$
- inertia of the arm: $J_a = 0.1$ kg.m^2

Here is the curve $\gamma(\tau) = \frac{\alpha_l}{\alpha_{opt}} = 2\frac{\tau}{1+\tau^2}$, where $\tau = \frac{r}{r_{opt}}$ with r – the reduction ratio and α – the acceleration.
(l stands for "load" and "opt" for "optimal")



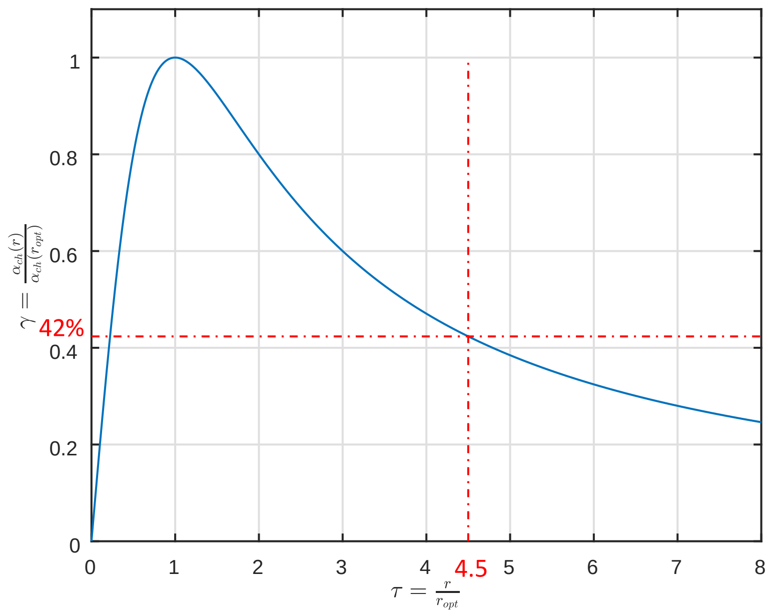
1. Determine the optimal reduction ratio for the considered segment.
2. With the given transmission in the question, determine the achievable rate of acceleration in relation with the optimal case.

Exercise 2 - Solution

1. The optimal reduction ratio is given by the expression:

$$r_{opt} = \sqrt{\frac{J_a}{J_m}}$$
$$= \sqrt{\frac{0.1\ kg.m^2}{625\ g.cm^2}}$$
$$= 40$$

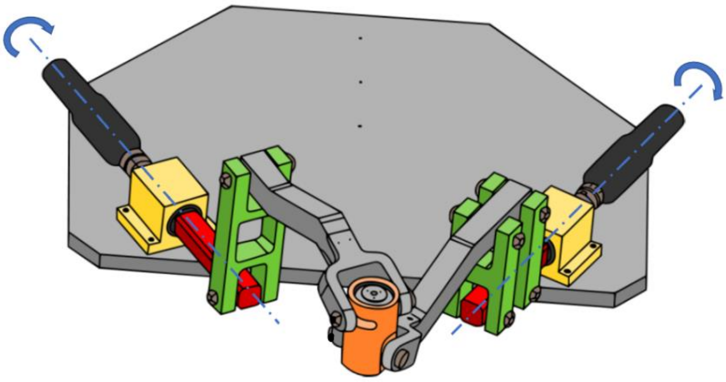
2. The used reduction ratio is 180. This ratio is 4.5 times larger than the optimal ratio.



From the curve $\gamma(\tau)$, we deduce that we only have 42% of the acceleration available when we use the optimal ratio.

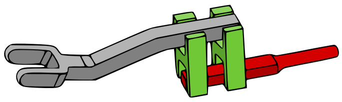
Exercise 3

Consider the robot shown below:

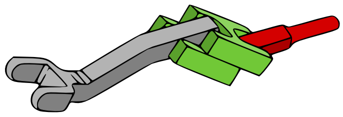


1. Give the type of architecture of the robot (serial or parallel).
2. Determine the number of DOFs.
3. Represent the vectors of the tool coordinates and the joint coordinates on the drawing.

One of the robot's "motor + reducer + arm + forearm" axes is shown below in two extreme postures. Each posture corresponds to a disposition of the robot and for each posture the equivalent inertia of the load (arm + forearm) is given by the value J_l.



(a) $J_l = 1.01 \times 10^{-1}$kg.m^2



(b) $J_l = 3.36 \times 10^{-2}$kg.m^2

Note: Although the geometry of the arms is not exactly the same on both sides, assume that they have the same equivalent inertia.

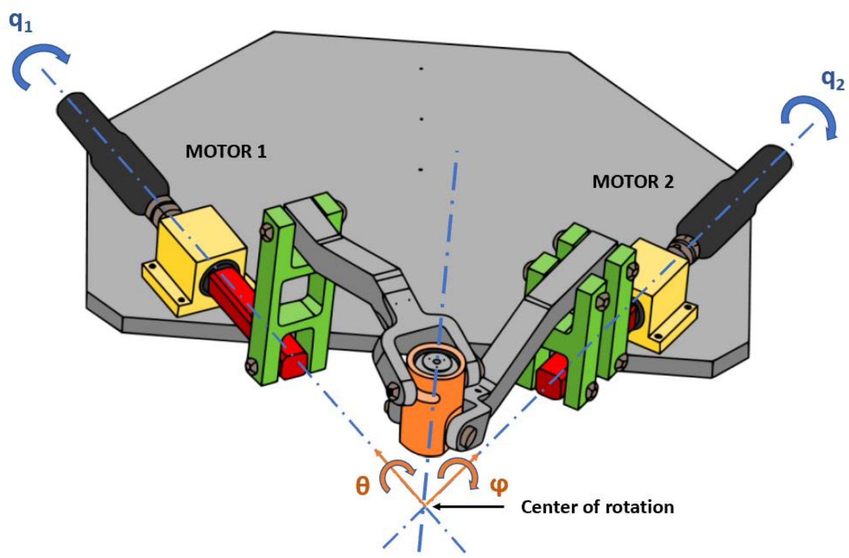
Knowing the following parameters:

- reduction ratio (GP62A): $n = 181$
- inertia of the reducer on the input side: $J_r = 88$ g.cm^2
- inertia of the motor (RE 50): $J_m = 542$ g.cm^2
- nominal motor torque: $\Gamma_m = 420$ mN.m

4. Give the equivalent moment of inertia referred to the output (load side).
5. Give the optimal reduction ratio for each configuration.
6. Are we too far from optimal performance?

Exercise 3 - Solution

1. It is a parallel robot because the kinematic chain is closed.
2. There are 2 rotational DOFs – see below an illustration showing the two tilts of the tool θ and ϕ. The center of rotation is the intersection of the two motors' axes.



3. Operational coordinates vector:

$$\boldsymbol{X} = \begin{pmatrix} \theta \\ \varphi \end{pmatrix}$$

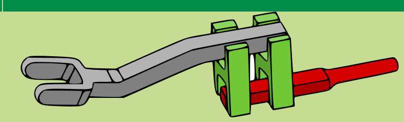
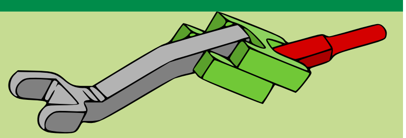
Joint coordinates vector (active joints):

$$\mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

4. The equivalent inertia referred to the output is given by:

$$J_{tot\_out} = J_l + (J_m + J_r)n^2$$

It is the total inertia referred to the output axis of each joint. It is variable depending on the joint positions q_1 and q_2 :

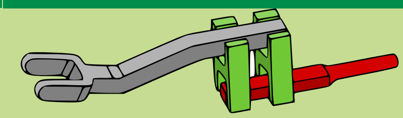
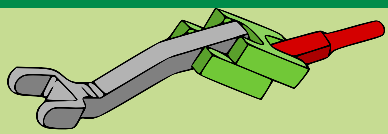
	Configuration A	Configuration B
		
J_l	1.01×10^{-1} kg.m²	3.36×10^{-2} kg.m²
J_{tot_out}	2.16 kg.m²	2.10 kg.m²

5. The optimal reduction ratio is given by:

$$r_{opt} = \sqrt{\frac{J_l}{J_m}}$$

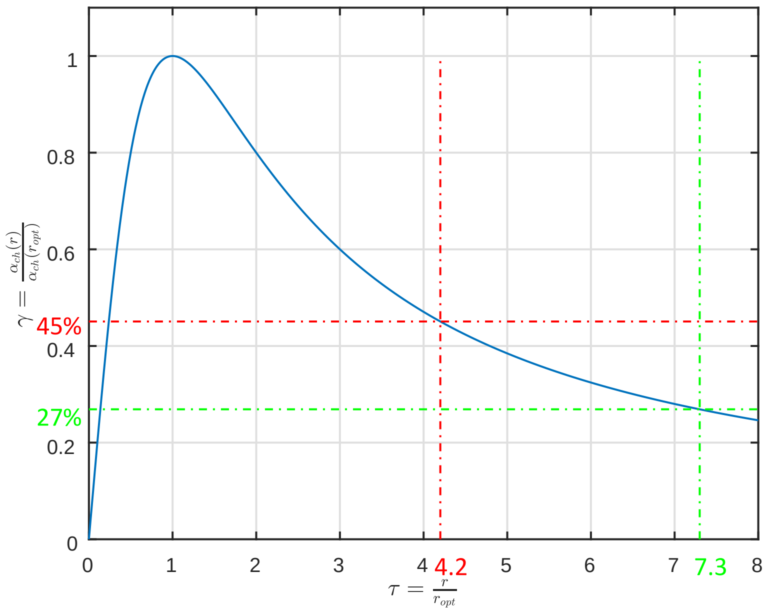
The moment of inertia of the reducer is not considered in the calculation because the goal is to size a reducer (unknown).

The following values are then obtained:

	Configuration A	Configuration B
		
J_l	$1.01 \times 10^{-1}\,\mathrm{kgm}^2$	$3.36 \times 10^{-2}\,\mathrm{kgm}^2$
r_{opt}	43.2	24.9
$r\,/\,r_{\text{opt}}$	4.2	7.3

6. The optimal ratio varies between 25 and 43. This means that the joint acceleration capacity is not the same depending on the working position of the robot.

Using the ratio $\frac{r}{r_{opt}}$ we can graphically calculate the achievable rate of acceleration in relation with the optimal case:



These rates are respectively 45% for configuration A and 27% for configuration B.